LCC '22 Contest 1 S3 - Candy Pathways

Time limit: 4.0s
Java: 6.0s
Python: 6.0s

Memory limit: 128M
Python: 256M

After escaping the flying eggs and finding used candy, you decide to do some trick-or-treating yourself! Your neighborhood has N houses with M roads connecting them. You live in house 1 and want to travel along these roads to reach your friend's house N. There are T types of candy and M roads, with each road i serving candy of type t_i. For every road i that you traverse, you will receive one candy of type t_i . Since you don't want cavities, you want to reach your friend's house while collecting **at most one** of each type of candy. How many distinct paths can you take from house 1 to house N while satisfying this constraint?

A few important things to note:

- **Since the answer can be quite large, please output the answer modulo** $10^9 + 7$.
- Your paths do not have to be simple (i.e. they may contain cycles).
- Once you reach house N, you may not traverse any more edges.
- Two paths are different if for some i, the i^{th} edge on one path is different from the i^{th} edge on the other.
- There may be multiple roads connecting the same pair of houses but no edge will connect the same house (e.g. $u_i \neq v_i$ holds true).
- The graph is not guaranteed to be connected.

Constraints

In all test cases, $2 \leq N \leq 18$, $1 \leq T \leq 18$, $1 \leq M \leq N^2$.

Note: You must solve all preceding subtasks in order to earn points for a specific subtask.

Subtask 1 [10%]

$N, T \leq 8$; in addition, the graph is guaranteed to form a tree (i.e. $M = N - 1$, no cycles exist, and the graph is connected).

Subtask 2 [20%]

$N, T \leq 8$

Subtask 3 [70%]

No further constraints.

Input Specification

First line: N, M, and T: the number of houses, number of roads, and number of candy types respectively.

Next M lines: u_i, v_i, and t_i ($1 \leq u_i, v_i \leq N$, $u_i \neq v_i$, $1 \leq t_i \leq T$), indicating a bidirectional road that connects houses u_i and v_i and serves candy of type t_i.

Output Specification

One integer: the number of distinct paths you can take while satisfying the described constraint **modulo** $10^9 + 7$.

Sample Input 1

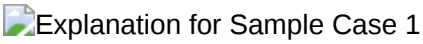
```
6 7 5
2 1 1
2 3 2
2 4 2
3 5 2
2 5 3
5 6 4
4 5 5
```

Output for Sample Case 1

```
2
```

Explanation for Sample Case 1

See a diagram of the neighbourhood below:


Explanation for Sample Case 1

The two ways to reach node N without receiving duplicate candies are

$1 \to 2 \to 5 \to 6$ and

$1 \to 2 \to 4 \to 5 \to 6$.

Sample Input 2

```
4 4 1
1 2 1
1 3 1
2 4 1
3 4 1
```

Output for Sample Case 2

```
0
```

Explanation for Sample Case 2

There is no possible sequence of roads you can traverse without receiving duplicate candy.